

Discrete Mathematics I

B. Math. II

Mid-Term Examination

Instructions: All questions carry ten marks. All graphs are assumed to be simple.

1. Let $n > 1$ be an integer. Prove that any n -regular graph with a cut edge must contain at least $2(n + 2)$ vertices.
2. Let $k > 1$ be an integer. Prove that every edge in a k -regular bipartite graph is contained in a cycle.
3. If (X_1, Y_1) and (X_2, Y_2) are minimum cuts in a transportation network, then prove that $(X_1 \cup X_2, Y_1 \cap Y_2)$ is also a minimum cut.
4. State True or False
 - (a) G is Hamiltonian if and only if it has no cut edge.
 - (b) The number of degree one vertices in a tree must be even.
 - (c) A 3-regular graph on eight vertices must contain a cut edge.
 - (d) If u, v are the only two vertices of odd degree in a graph G , then G contains a $u - v$ path.
 - (e) A tree has at most one perfect matching.